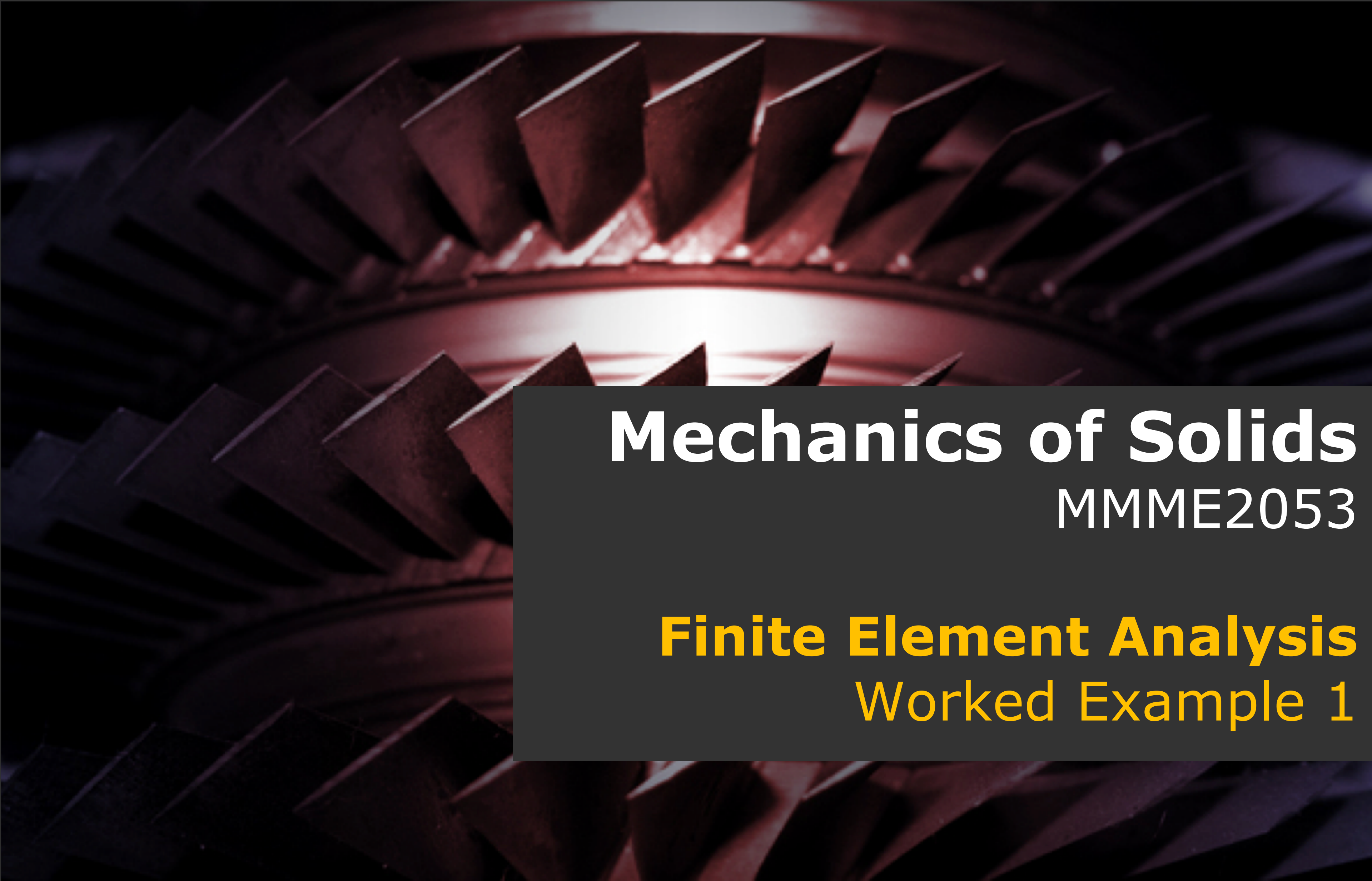


Mechanics of Solids

MMME2053

Finite Element Analysis

Worked Example 1

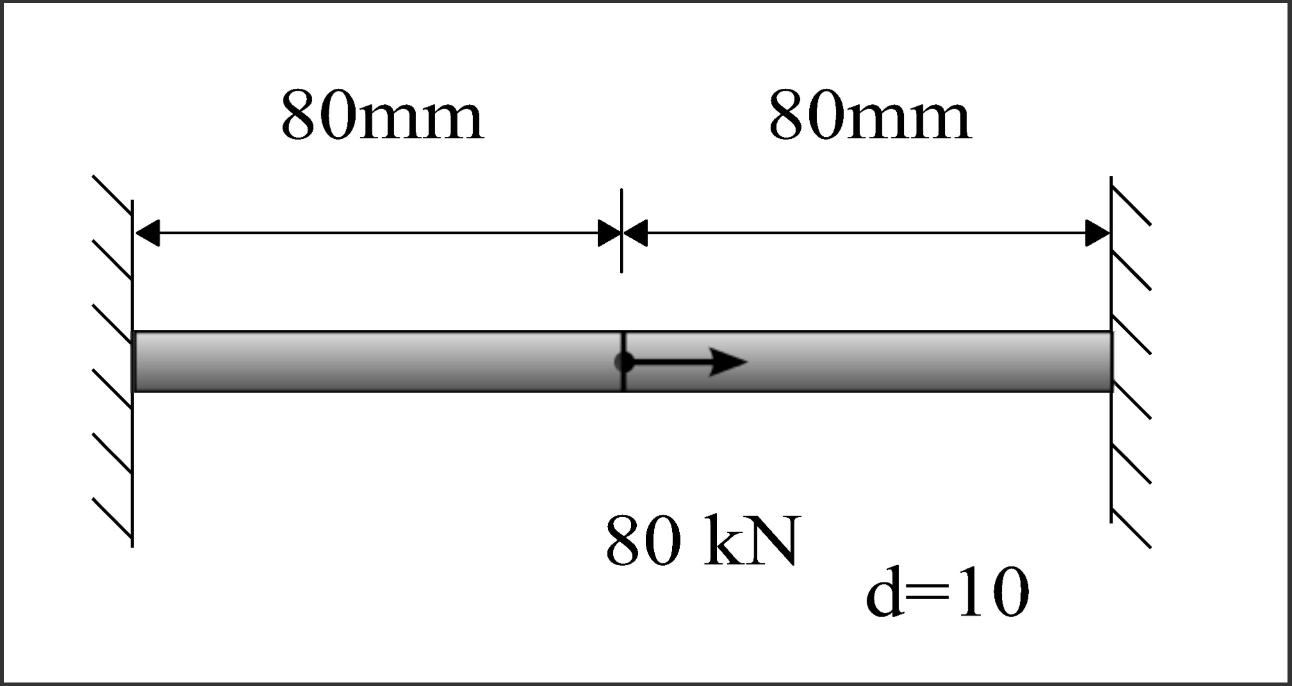
Application of Spring Elements

- Now that we have a system of equations in matrix form that define the relationships between the forces and displacements in our system, we can start to use it to solve problems.

- If we subjects the system to a force(s) and sufficient boundary conditions are specified, the remaining forces and displacements can be found.

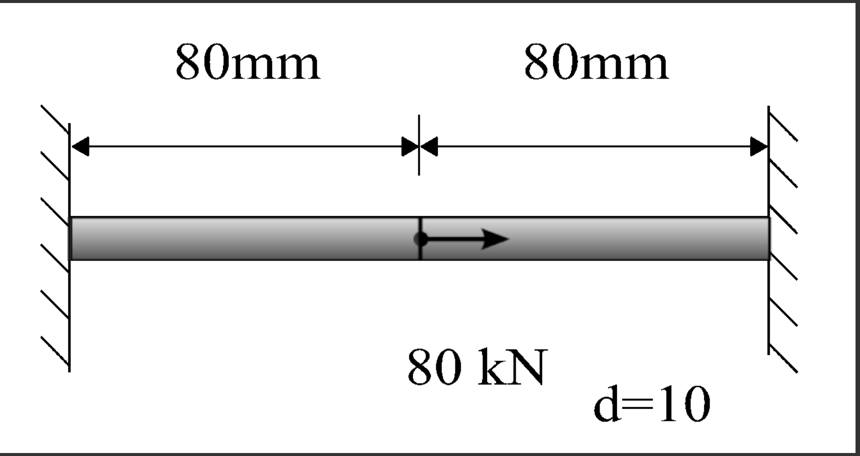
Application of Spring Elements

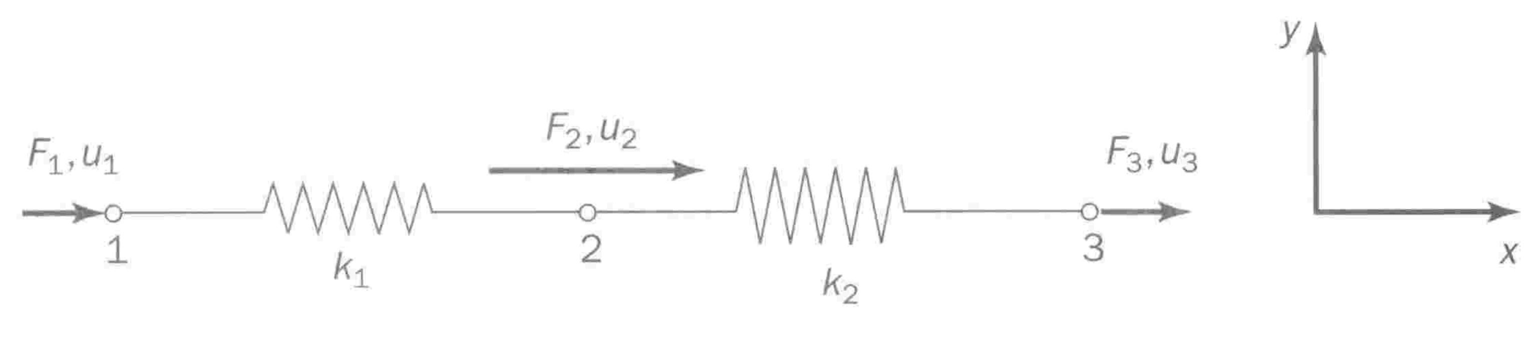
- Two steel rods are connected together and loaded at the connection, determine the displacement of the point where the load is applied: (*E* = 200 GPa)



Application of Spring Elements

- If sufficient boundary conditions are specified, the forces and displacements can be found.





Application of Spring Elements

1. Determine *AE/L*

2. Construct the element stiffness matrices

3. Combine to form the global stiffness matrix

4. Apply the boundary conditions

5. Solve for displacements

Application of Spring Elements

1. Determine AE/L

$A = \pi r^2$

$A = \pi \times (5\times10^{-3})^2 = 7.8540 \times 10^{-5}\ \mathrm{m}^2$

$E = 200 \times 10^{9}\ \mathrm{N/m}^2$

$L = 80 \times 10^{-3}\ \mathrm{m}$

$k = AE/L = 1.9635 \times 10^{8}\ \mathrm{N/m}$

Application of Spring Elements

2. Construct the element stiffness matrices

- Element 1:

$$[K_e]_1 = \begin{bmatrix} 1.9635\times10^8 & -1.9635\times10^8 & 0 \\ -1.9635\times10^8 & 1.9635\times10^8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Element 2:

$$[K_e]_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.9635\times10^8 & -1.9635\times10^8 \\ 0 & -1.9635\times10^8 & 1.9635\times10^8 \end{bmatrix}$$

Application of Spring Elements

3. Combine to form the global stiffness matrix

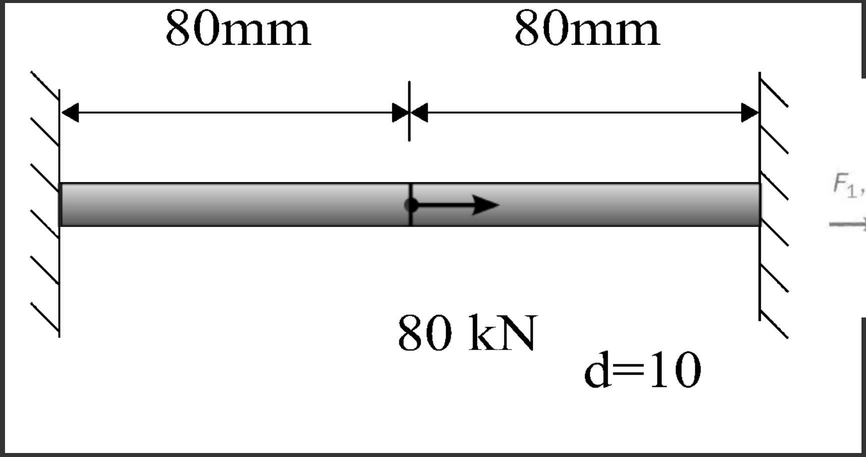
e1

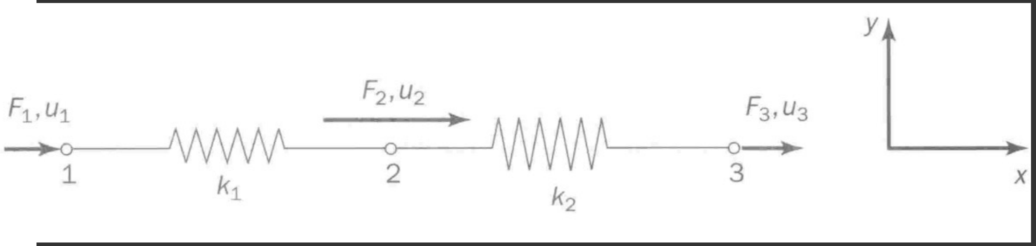
$$[K] = \begin{bmatrix} 1.9635\times10^8 & -1.9635\times10^8 & 0 \\ -1.9635\times10^8 & 1.9635\times10^8 + 1.9635\times10^8 & -1.9635\times10^8 \\ 0 & -1.9635\times10^8 & 1.9635\times10^8 \end{bmatrix}$$

e2

Application of Spring Elements

4. Apply the boundary conditions





$u_1 = u_3 = 0$

$F_2 = 80000$ N

Application of Spring Elements

4. Apply the boundary conditions

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 1.9635\times10^8 & 1.9635\times10^8 & 0 \\ -1.9635\times10^8 & 1.9635\times10^8 + 1.9635\times10^8 & -1.9635\times10^8 \\ 0 & 1.9635\times10^8 & 1.9635\times10^8 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$F_2 = 3.9270\times10^8 u_2$$

Application of Spring Elements

5. Solve for the displacement

$$80000 = 3.9270\times10^{8}u_2$$

$$u_2 = \frac{80000}{3.9270\times10^{8}}$$

$$u_2 = 2.0372\times10^{4}\ \mathrm{m} = 0.203\ \mathrm{mm}$$

Learning Objectives

1. Recognise that FEA is a useful technique to aid the solution of many Structural Mechanics problems
2. Understand how 1D elements and the matrix method can be used to analyse uniaxial bars
3. Apply theory for 1D elements and the matrix method to an assembly of bars
4. Understand the derivation of the global stiffness matrix of a truss element